Lecture 5

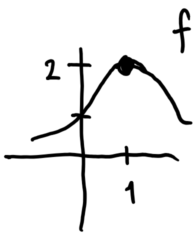
Tuesday, September 6, 2016 8:52 AM

<u>Recall</u>

A function f is continuous at a if

$$\lim_{x \to a} f(x) = f(a)$$

<u>Ex</u>



$$\lim_{x \to 1} f(x) = 2$$

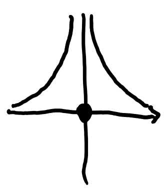
$$f(1) = 2$$

$$\lim_{x \to 1} f(x) \neq f(1)$$

Therefore, f is discontinuous at 1.

Removable discontinuity.

<u>Ex 2</u> $f(x) = \begin{cases} \frac{1}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

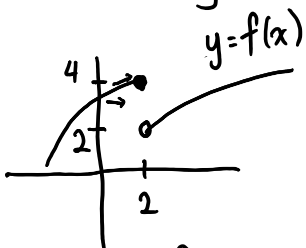


$$\lim_{x \to 0} f(x) = \infty$$

$$f(0) = 0$$

Infinite discontinuity at $x = 0$.

<u>Ex 3</u>



$$\lim_{x \to 2^-} f(x) = 4 = f(2) = 4$$

$$\lim_{x \to 2^+} f(x) = 2$$

$$\lim_{x \to 2} f(x) \text{ DNE}$$

Continuous from

left (at a

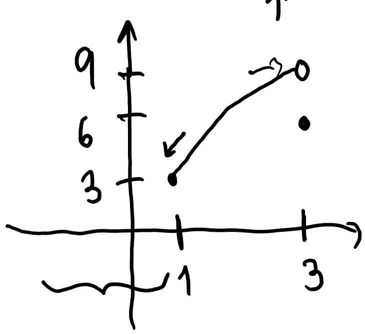
Jump Discontinuity at $x = 2$.

Def A function f is said to be continuous at a from the right if $\lim_{x \to a^+} f(x) = f(a)$
& f is continuous from the left at a if $\lim_{x \to a^-} f(x) = f(a)$

Def A function f is continuous on an interval if it is continuous at every point in the interval.

Rmk If a function is defined on only one side of an endpoint of an interval, it is understood that continuity at an endpoint means left or right continuity.

Ex f defined on $[1,3]$ including



$(1,3)$ not included

For f to be continuous @ 1,

$$\underline{\lim_{x \to 1^+} f(x)} = f(1)$$

f is contin @ 1

For f to be continuous @ 3

$$\lim_{x \to 3^-} f(x) = f(3)$$

$$\begin{array}{ccc} \lim_{x\to 3^-} f(x) & & f(3) \\ \shortparallel & & \shortparallel \\ 9 & \neq & 6 \end{array}$$

f is cont. on $[1,3)$

<u>Theorem</u> f, g are cont. @ a and c is a constant. Then the following funcs are cont. @ a

1) $f \pm g$ 2) fg ← product 3) cf

4) $\frac{f}{g}$ if $g(a) \neq 0$ (Limit Laws) DIY

<u>Thm</u> 1) A polynomial is continuous everywhere.

2) The following type of funcs are continuous at every number in the domain:

polynomial, rational, root, trigonometric, inverse trig., exponential, logarithmic fns.

Thm If f is continuous at b,

$\lim\limits_{x\to a} g(x) = b$, then $\lim\limits_{x\to a} f(g(x)) = f(b)$

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$$

Ex $\lim\limits_{x\to 1} \arccos\left(\dfrac{1-\sqrt{x}}{1-x}\right)$

Since arccos is continuous

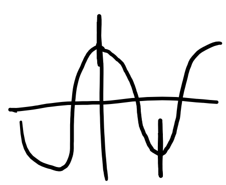
$$= \arccos\left(\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x}\right) \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}}$$

$a^2 - b^2 = (a-b)(a+b)$

$$= \arccos\left(\lim_{x\to 1} \frac{\cancel{(1-\sqrt{x})}}{\cancel{(1-\sqrt{x})}(1+\sqrt{x})}\right)$$

$$= \arccos\left(\lim_{x\to 1} \frac{1}{1+\sqrt{x}}\right)$$

$$= \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$$



Thm If g is continuous at a,

f is continuous at $g(a)$,

then $f\circ g$ is continuous at a.

Intermediate Value Theorem

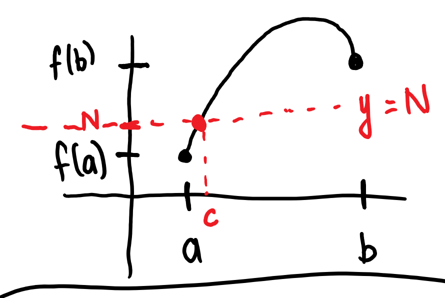
Suppose f is continuous on a closed interval $[a,b]$, and $f(a) \neq f(b)$.

Let N be any number between $f(a)$ and $f(b)$.

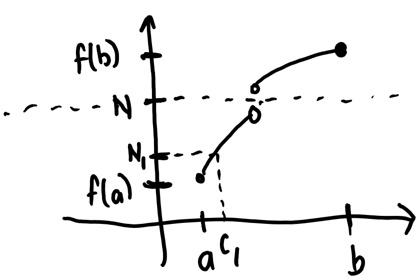
Then there exists a number c between a and b (i.e $c \in (a,b)$) such that $f(c) = N$.

Rmk A continuous function takes on every value between $f(a)$ and $f(b)$.

Graphically



Continuity Here is Vital



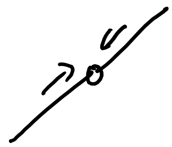
There is no c betn a & b such that $f(c) = N$

HH 407
11-12 pm today

$\frac{1}{x-2}$ $x \neq 2$

continuous $\rightarrow (-\infty, 2) \cup (2, \infty)$

con $(-\infty, a) \cup (a, \dots$



@ b

f cont, $\lim_{x \to a} g(x) = b$.

$$\lim_{x \to a} f(g(x)) = f(b)$$

1

$\cos(x^2+1)$

$f(x) = \cos x$

$g(x) = x^2 + 1$

$f \circ g = \cos(x^2+1)$